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THE ROLE OF PDE IN TRANSONIC AERODYNAMICS AND MORPHING WING DESIGN : A SYSTEMATIC LITERATURE REVIEW

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Abstract. Fluid dynamics and aerodynamic design of aircraft wings play a crucial role in improving flight efficiency, especially in the transonic regime. This research aims to analyze the application of partial differential equations (PDE) in aircraft design optimization and explore technical solutions to challenges that arise in Computational Fluid Dynamics (CFD). Our research method is Literature Review (SLR) using systematic steps such as database search, screening, and thematic analysis based on references from recent scientific journals from 2020 to present (2025) through ScienceDirect, Google Scholar, arXiv, MDPI, DOAJ, and Academia. An initial search yielded 926 journals on PDEs, but after selection, 36 journals met the inclusion criteria and were thematically analyzed. The main findings show that the Navier-Stokes, Continuity, and Bernouli Equations are the cornerstones in understanding the fluid flow behavior around an Aircraft wing. Advances in CFD technology have led to innovation, such as the application of morphing wings and winglets, which can improve aerodynamic efficiency and reduce fuel consumption. However, challenges such as CFD solver robustness, design complexity with many variables, and geometry constraints are still an obstacle in aerodynamic optimization. It can be concluded that utilizing PDE in CFD contributes significantly to the development of higher efficiency aircraft and supports design innovations that are more adaptive to modern aerodynamic demands, especially in the transonic regime.

Keywords: Adaptive wing design; Aerodynamic design; Computational Fluid Dynamics (CFD); Digital laboratory optimization; Partial Differential Equations; Regime Transonic: Systematic literature review.

***Abstrak**. Dinamika fluida dan desain aerodinamika sayap pesawat memainkan peran penting dalam meningkatkan efisiensi penerbangan, terutama dalam regime transonik. Penelitian ini bertujuan untuk menganalisis penerapan persamaan diferensial parsial (PDE) dalam optimasi desain pesawat dan mengeksplorasi solusi teknis terhadap tantangan yang muncul dalam Computational Fluid Dynamics (CFD). Metode penelitian kami adalah Tinjauan Pustaka Sistematis (SLR) menggunakan langkah-langkah sistematis seperti pencarian database, penyaringan, dan analisis tematik berdasarkan referensi dari jurnal ilmiah terbaru dari tahun 2020 hingga saat ini (2025) melalui ScienceDirect, Google Scholar, arXiv, MDPI, DOAJ, dan Academia. Pencarian awal menghasilkan 926 jurnal mengenai PDE, namun setelah seleksi, 36*

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jurnal memenuhi kriteria inklusi dan dianalisis secara tematik. Temuan utama menunjukkan bahwa Persamaan Navier-Stokes, Persamaan Kontinuitas, dan Persamaan Bernoulli adalah dasar dalam memahami perilaku aliran fluida di sekitar sayap pesawat. Kemajuan dalam teknologi CFD telah menghasilkan inovasi, seperti penerapan sayap morfing dan winglet, yang dapat meningkatkan efisiensi aerodinamis dan mengurangi konsumsi bahan bakar. Namun, tantangan seperti ketahanan solver CFD, kompleksitas desain dengan banyak variabel, dan keterbatasan geometri masih menjadi hambatan dalam optimisasi aerodinamis. Dapat disimpulkan bahwa pemanfaatan PDE dalam CFD memberikan kontribusi signifikan terhadap pengembangan pesawat dengan efisiensi lebih tinggi dan mendukung inovasi desain yang lebih adaptif terhadap tuntutan aerodinamis modern, terutama dalam rezim transonik.

Kata kunci: *Desain sayap adaptif; Desain aerodinamis; Computational Fluid Dynamics (CFD); Optimisasi laboratorium digital; Persamaan Diferensial Parsial; Rezim Transonik: Tinjauan pustaka sistematis.*

1. Introduction

Computational Fluid Dynamics (CFD) has become a fundamental tool in analyzing and optimizing fluid flow behavior in aerodynamic systems, particularly in aircraft wing design [1]. In CFD analysis, solving the Navier-Stokes equations is crucial because it allows understanding of fluid behavior. At the core of CFD are Partial Differential Equations (PDEs), such as the Navier-Stokes equations, which govern fluid motion and enable the prediction of complex flow phenomena under various operating conditions [2]. Recent studies also highlight the increasing role of PDE-constrained optimization in improving aerodynamic performance and design efficiency [3].

The accurate numerical solution of these equations is essential for understanding aerodynamic performance, especially in regimes where flow behavior becomes highly nonlinear. In the transonic regime (Mach 0.8-1.2), aerodynamic analysis becomes significantly more challenging due to the coexistence of subsonic and supersonic flow regions [4]. This condition leads to complex phenomena such as shock wave formation, boundary layer interaction, and flow separation, which directly affect lift, drag, and overall aircraft performance. To address these challenges, advanced CFD techniques based on PDE formulations are required to model and simulate fluid behavior with high accuracy [5].

Recent developments in adaptive and morphing wing technologies have further increased the importance of PDE-based modeling, particularly with the integration of advanced CFD solvers and hybrid numerical approaches that enhance aerodynamic efficiency and simulation accuracy [32], [33]. These designs allow real-time geometric adaptation of the wing to optimize aerodynamic efficiency under varying flight conditions [6]. However, the integration of PDE-based numerical methods with adaptive design introduces additional complexity, particularly in terms of computational cost, solver robustness, and multi-physics coupling, such as aeroelastic interactions [7]. Despite the growing number of studies in this area, there remains a lack of systematic synthesis that clearly explains how PDEs are applied within CFD frameworks for transonic aerodynamic optimization and morphing wing design [8]. Most existing studies focus on specific implementations or individual numerical approaches without providing a comprehensive comparison of methods, trends, and research gaps.

Therefore, this study employs a Systematic Literature Review (SLR) approach to analyze and synthesize recent research on the role of PDEs in CFD, particularly in the context of transonic aerodynamics and adaptive wing design. This review aims to (1) identify the dominant PDE models and numerical methods used, (2) compare their effectiveness in aerodynamic optimization, and (3) highlight current research gaps and future development opportunities.

2. Research Methods

2.1 Research Design

This study employs a Systematic Literature Review (SLR) approach to analyze the role of Partial Differential Equations (PDEs) in Computational Fluid Dynamics (CFD), particularly in transonic aerodynamics and adaptive wing design. The SLR method is used to systematically identify, evaluate, and synthesize relevant research findings in a transparent and reproducible manner.

2.2 Search Strategy

The literature search was conducted using several major academic databases, including ScienceDirect, Scopus, Web of Science, IEEE Xplore, and DOAJ. These databases were selected due to their credibility, comprehensive coverage, and availability of peer-reviewed scientific articles relevant to CFD and PDE research. The search process used a combination of keywords and Boolean operators to ensure comprehensive coverage of relevant studies. The main search query was formulated as follows: ("Partial Differential Equation" OR "PDE") AND ("Computational Fluid Dynamics" OR "CFD") AND ("Transonic Flow" OR "Transonic Aerodynamics") AND ("Morphing Wing" OR "Adaptive Wing") The search was limited to articles published between 2020 and 2025 to ensure the inclusion of recent developments in the field.

2.3 Inclusion and Exclusion Criteria

The selection of articles was based on predefined inclusion and exclusion criteria to ensure relevance and quality. The criteria are summarized as follows:

Inclusion criteria:

- Peer-reviewed journal articles
- Published between 2020 and 2025
- Written in English
- Focus on PDE applications in CFD, transonic flow, or adaptive wing design
- Include numerical methods or computational analysis

Exclusion criteria:

- Non-peer-reviewed sources (e.g., blogs, opinion articles, informal proceedings)
- Articles not directly related to PDE or CFD
- Studies without numerical or computational analysis
- Duplicate publications

2.4 Screening Process

The article selection process followed the PRISMA (Preferred Reporting Items for Systematic Reviews and Meta-Analyses) framework. In the initial stage, all identified articles were collected from the selected databases. Duplicate records were removed prior to screening.

In the screening stage, titles and abstracts were reviewed to identify studies that met the inclusion criteria. Articles that were not relevant to PDE applications in CFD or transonic aerodynamics were excluded. In the eligibility stage, full-text articles were assessed to ensure their methodological quality and relevance. Studies that did not provide sufficient methodological details or did not focus on numerical approaches were excluded. Finally, a total of 36 articles were selected for further analysis and synthesis.

2.5 Data Extraction and Analysis

Data from the selected articles were systematically extracted, including information on research objectives, PDE models used, numerical methods, simulation approaches, and key findings. The extracted data were analyzed using a thematic synthesis approach. The studies were grouped based on common themes, such as:

- PDE-based flow modeling
- Numerical methods in CFD
- Aerodynamic optimization techniques
- Adaptive and morphing wing applications

This approach enables the identification of research trends, methodological differences, and existing gaps in the literature.

Table 1. Inclusion and exclusion criteria for article selection

Criteria	Equalization	Exception
Document type	Peer-reviewed and officially indexed journal article	Non-peer-reviewed articles such as informal proceedings, blog posts, and opinion articles.
Year of publication	2020-2025	Outside this range
Language	English	Languages other than english
Research topic	Fluid dynamics, adaptive wing design, transonic regime, partial differential equations	Fluid dynamics, adaptive wing design, transonic regime, partial differential equations
Research focus	Numerical study, CFD simulation, adaptive wing design optimization	Studies that do not involve numerical analysis or adaptive design

From the initial search of 926 articles using the keyword "Partial Differential Equation in Transonic Aerodynamics," screening was conducted based on year (2020-2025), title, and inclusion criteria. As a result, 36 relevant articles were successfully analyzed. Each article was extracted based on its identity, purpose, method, and sample, then thematically grouped and descriptively analyzed to answer the research questions. The selection process is visualized through a PRISMA diagram in Figure 1.

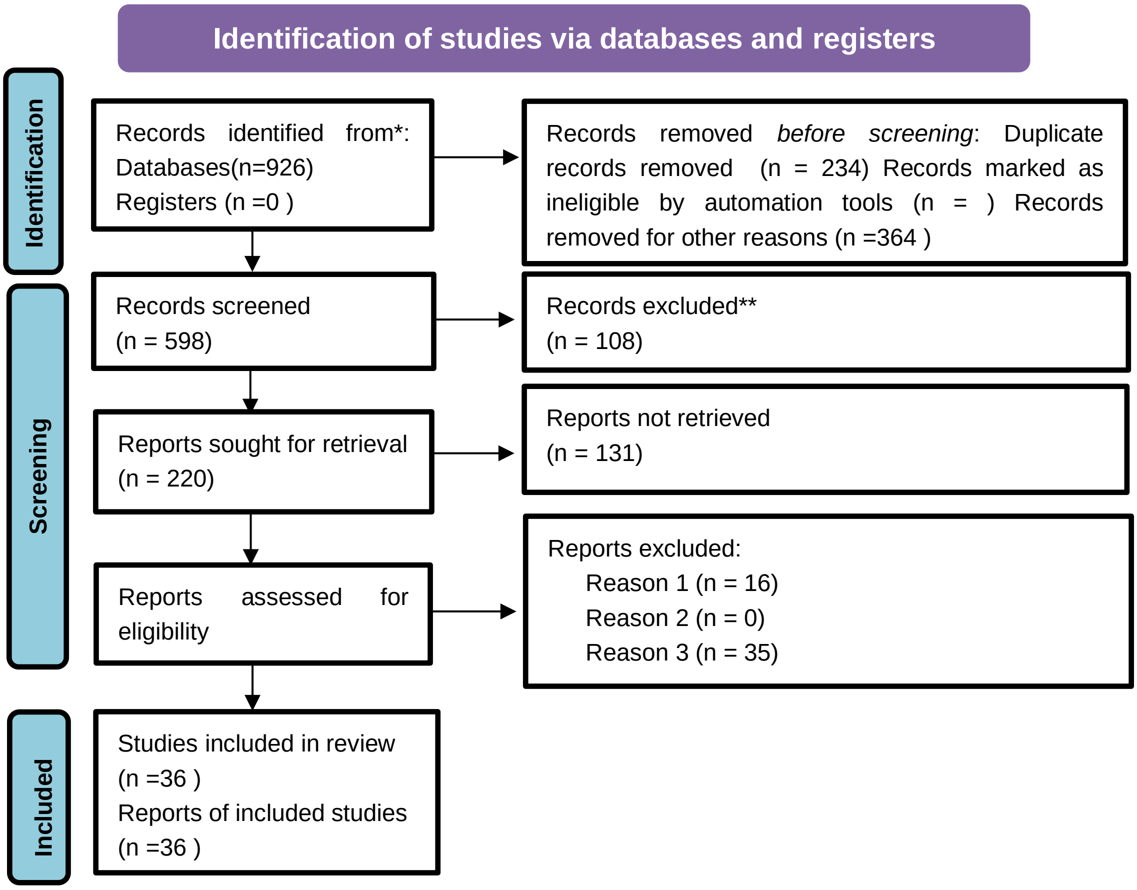

Figure 1. PRISMA flow diagram

Table 2. Summary of Characteristics of Analyzed Articles

No.	Article	Summary
1.	[1]	Transonic flow simulation; Method: Finite Volume; PDE: Navier–Stokes; Innovation: Adaptive control surfaces; Findings: Increased stability at Mach 0.85.
2.	[9]	Drag optimization; Method: RANS + LES hybrid; PDE: NS time-dependent; Innovation: Morphing wing; Findings: Drag reduced by 18%.
3.	[6]	Aeroelastic interaction; Method: FSI; PDE: Euler + Structural PDE; Innovation: Flexible wing; Findings: Real-time control is necessary.
4.	[7]	Solver convergence; Method: Spectral; PDE: Compressible NS; Findings: Unstable in complex geometries.

3. Result and Discussion

3.1 Overview of Selected Studies

A total of 36 articles were selected and analyzed based on the predefined inclusion and exclusion criteria. The selected studies primarily focus on the application of Partial Differential Equations (PDEs) in Computational Fluid Dynamics (CFD), particularly in transonic aerodynamics and adaptive wing design. Most of the studies employ the Navier-Stokes equations as the fundamental PDE model for fluid flow simulation [1], [7]. In addition, several studies incorporate coupled systems, such as PDE-ODE models, to represent aeroelastic interactions in flexible and morphing wing structures [10].

These approaches highlight the central role of PDEs in modeling complex aerodynamic phenomena. In terms of numerical approaches, the majority of studies utilize the Finite Volume Method (FVM), while others apply hybrid approaches such as RANS-LES models [11-12], as well as spectral methods for specific cases [7]. These variations indicate the diversity of computational strategies used to solve PDEs in aerodynamic simulations.

3.2 Thematic Synthesis of PDE Applications in CFD

Based on the analysis of the selected studies, three major themes were identified regarding the application of PDEs in CFD

a. PDE for Flow Modeling

PDEs, particularly the Navier–Stokes equations, are widely used to model fluid flow behavior around aircraft wings [1], [13]. These equations enable the representation of velocity, pressure, and turbulence effects under transonic conditions. The studies consistently show that accurate PDE-based modeling is essential for capturing shock wave formation and flow separation [14][15].

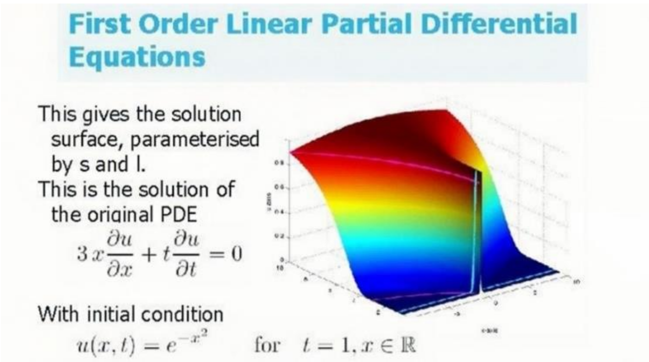

Figure 1. First Order Partial Differential Equations [16]

b. PDE for Aerodynamic Optimization

Several studies apply PDE-based methods in aerodynamic shape optimization. By integrating PDE solvers with optimization algorithms, researchers are able to minimize drag and improve lift-to-drag ratios [17], [18]. This approach is particularly effective in transonic regimes, where small geometric changes significantly impact aerodynamic performance, and recent research demonstrates that PDE-constrained optimization frameworks can further enhance design adaptability and computational efficiency [3]

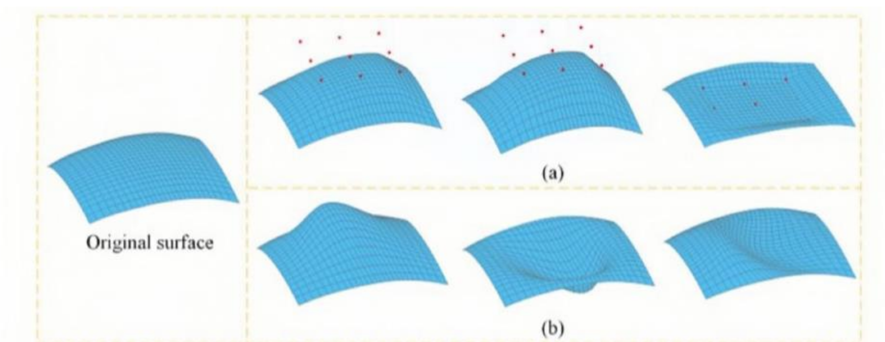

Figure 2. A comparison between the free-form deformation and PDE-based modeling methods. a The free-form deformation with 9 red control points which are governed by one design variable. b PDE-based deformation by changing the value of 3 control parameters (3 design variables) [19]

c. PDE in Aeroelastic and Morphing Wing Systems

PDEs are also used to model the interaction between aerodynamic forces and structural deformation. In morphing wing systems, PDE-based models are coupled with structural equations to simulate real-time shape adaptation [20] [21]. This integration allows for improved performance but also introduces higher computational complexity, and recent studies further confirm that coupled PDE-based aeroelastic modeling is essential for the development of next-generation morphing wing systems [22].

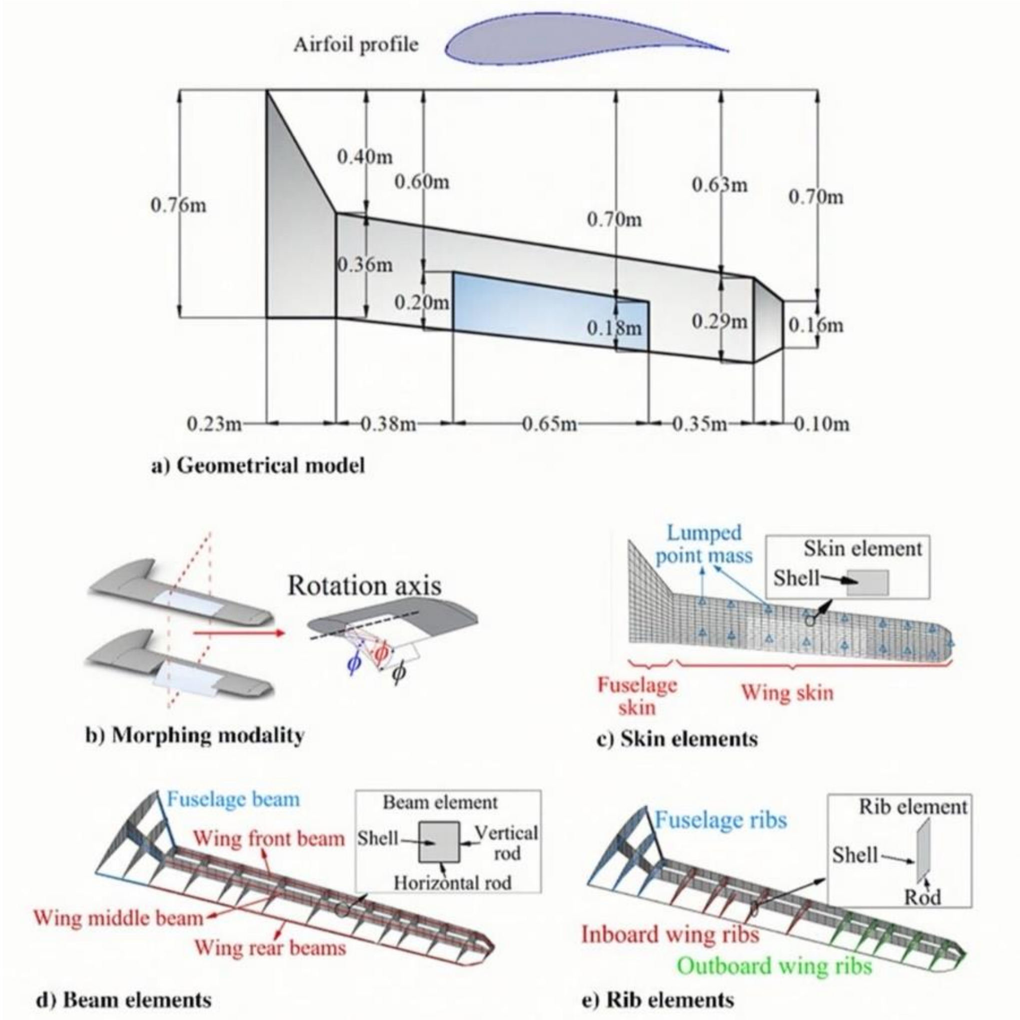

Figure 3. Structural design of the morphing wing [23]

This image displays the structural design of a morphing wing. The first part shows a geometric airfoil model with detailed dimensions, which serves as the basis for the wing shape. Next, the morphing modality diagram illustrates the wing shape-changing mechanism through a rotation axis. Then, there is an illustration of skin elements showing the fuselage and wing skin with distributed mass, followed by beam elements that display the main frame consisting of front, middle, and rear beams. Finally, rib elements show the arrangement of fuselage ribs, inner wing, and outer wing ribs. Overall, this image explains how the combination of skin, beam frame, and ribs forms a morphing wing structure that can change shape to adapt to aerodynamic conditions.

3.3 Comparison of Numerical Methods

The reviewed studies reveal significant differences in the numerical methods used to solve PDEs in CFD.The Finite Volume Method (FVM) is the most commonly used approach due to its stability and suitability for complex geometries [9], supported by recent advancements in efficient Navier–Stokes solvers that enable high-fidelity simulations in complex aerodynamic conditions [36]. it may require finer meshes to achieve high accuracy. In contrast, spectral methods offer high accuracy for smooth problems but tend to be less stable in complex geometries and nonlinear conditions [26]. Nevertheless, both methods can be used to solve problems in solid mechanics as well as in fluid mechanics, since both are numerical methods for the approximate solution of partial differential equations [25]

Tabel 3. Comparison of Numerical Methods in PDE-based CFD Studies

No	Study	PDE Model	Numerical Method	Application	Advantages	Limitations
1	[1]	Navier-Stokes	Finite Volume Method (FVM)	Transonic flow simulation	Stable for complex geometry	Requires fine mesh for accuracy
2	[6]	Navier-Stokes	Spectral Method	Flow analysis	High accuracy for smooth flow	Less stable for shock waves
3	[8]	Navier-Stokes	RANS-LES Hybrid	Turbulence modeling	Balance between cost & accuracy	Complex implementation
4	[24]	PDE-ODE Coupling	FEM+CFD	Morphing wing	Captures aeroelastic interaction	High computation cost
5	[25]	Navier-Stokes	Optimization-based CFD	Aerodynamic optimization	Improves lift-to-drag ratio	Sensitive to intial conditions

Hybrid methods, such as RANS–LES models, provide a balance between computational cost and accuracy, making them suitable for turbulence modeling [9]. Overall, no single method is universally superior; the choice depends on the problem characteristics and computational requirements.

In the case of the diffuser flow (Figure 4), the boundary layer moves axially from the inner cylinder. The outward-expanding slip wall generates a pressure gradient, causing the boundary layer to become much thicker. This test is not intended to assess the accuracy of the RANS model, but rather to show how the protective function operates differently and how it affects the distribution of eddy viscosity within the boundary layer.

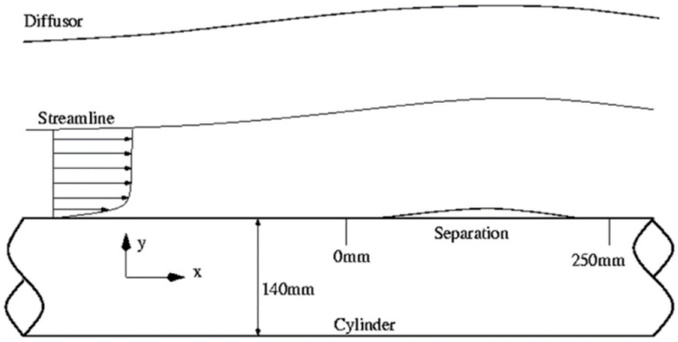

Figure 4. Computational domain for the CS0 axisymmetric diffuser [27]

3.4 Role of PDE in Transonic Aerodynamics

PDEs play a fundamental role in understanding aerodynamic behavior in the transonic regime, where flow characteristics become highly nonlinear [28], supported by recent CFD advancements that improve simulation accuracy under complex and nonlinear flow conditions [29], [30]. The presence of shock waves, compressibility effects, and flow instability requires accurate mathematical modeling based on PDE formulations.

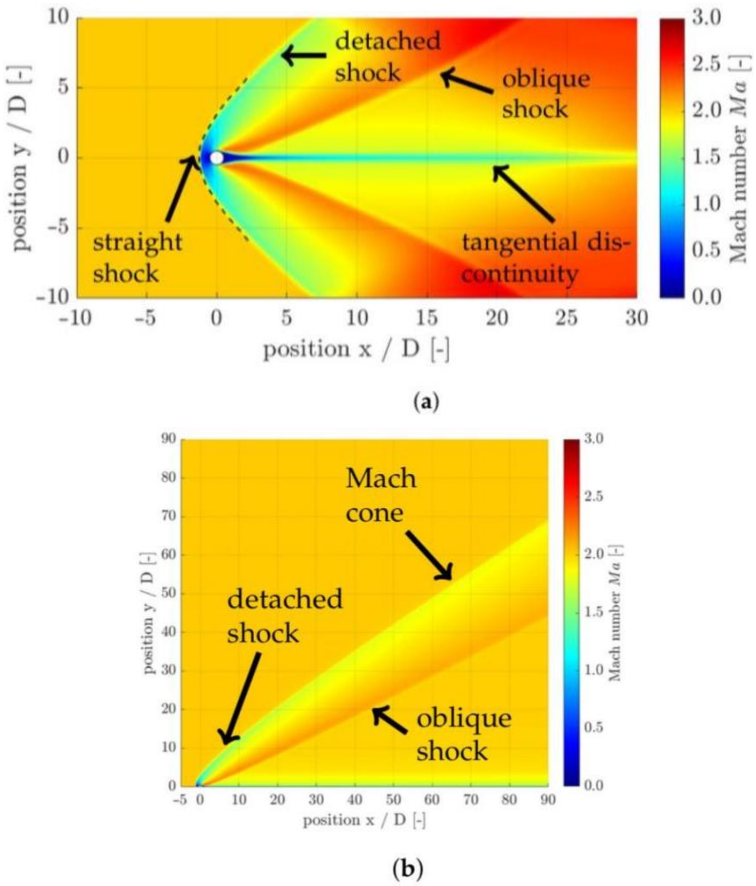

Figure 2. Transient results of velocity in the x direction x (ux) at ReD=100 and Ma∞=0,2 with a time step size of 10-8 s (a) “High Resolution” advection scheme; (b) “Upwind” advection scheme [11].

Flow simulations show that the shape and boundaries of the domain greatly affect shock wave patterns and velocity distribution. When the flow speed is high, shock waves form, causing pressure surges and flow discontinuities [11]. The Navier–Stokes equations are essential for capturing these phenomena, while simplified models such as Euler equations are sometimes used to reduce computational cost [31]. The studies indicate that the accuracy of transonic simulations strongly depends on the effectiveness of numerical methods in solving these PDEs.

3.5 Research Gaps

Despite significant progress, several research gaps were identified, particularly in improving computational efficiency and achieving real-time optimization in PDE-based CFD simulations, despite recent advancements in numerical methods and solver development [30], [32]. First, there is a lack of real-time PDE-based optimization methods for morphing wing systems under dynamic flight conditions [33]. Second, the coupling between aerodynamic and structural models remains computationally expensive and complex to implement [24]. Third, many studies rely heavily on numerical simulations without sufficient experimental validation, limiting the reliability of the results [34]. Finally, there is limited research on improving computational efficiency while maintaining high accuracy in transonic CFD simulations.

3.6 Challenges in Practical Implementation

Several challenges remain in the practical application of PDE-based CFD methods. One major challenge is the robustness of CFD solvers when dealing with complex geometries and nonlinear boundary conditions [35]. In addition, the large number of design variables in aerodynamic optimization increases computational complexity [5] Geometric constraints also limit the flexibility of design optimization, while high computational cost remains a significant barrier, particularly for high-fidelity simulations [36].

4. Conclusion

Based on the results of a systematic literature review, the use of partial differential equations in fluid dynamics and aircraft aerodynamic design has contributed significantly to flight efficiency. Through CFD simulations, innovations such as morphing wings and biomimetic structures have improved aerodynamic performance. Although there are challenges in the application of PDP, such as geometric complexity and high computational requirements, aerostructural optimization-based approaches and numeric methods have shown potential as effective solutions. Further development of optimization techniques and integration of data-based methods are expected to accelerate the engineering cycle and improve aircraft aerodynamic efficiency more broadly. This research aims to explore the role of PDP in improving modern aircraft design and identify technical solutions that can overcome these challenges. Based on a literature synthesis, PDP-based approaches have proven to be an important foundation in methods are expected to accelerate the engineering cycle and improve aircraft aerodynamic efficiency more broadly. This research aims to explore the role of PDP in improving modern aircraft design and identify technical solutions that can overcome these challenges. Based on a literature synthesis, PDP-based approaches have proven to be an important foundation in airflow modeling and aerodynamic optimization, and this is further supported by recent advancements in PDE-constrained optimization and high-efficiency numerical solvers that continue to push the boundaries of modern aerodynamic design.

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References

1. S. S. Wang, F. Z., Animasaun, I. L., Muhammad, T., & Okoya, "Recent advancements in fluid dynamics: Drag reduction, lift generation, computational fluid dynamics, turbulence modelling, and multiphase flow," Arab. J. Sci. Eng., vol. 49, no. 8, pp. 10237–10249., 2024, doi: https://doi.org/10.1007/s13369-024-08945-3.
2. S. Zhang, Z., Wang, Q., & Zhang, "Review of computational fluid dynamics analysis in biomimetic applications for underwater vehicles.," Biomimetics, vol. 9, no. 2, p. 79, 2024, doi: https://doi.org/10.3390/biomimetics9020079.
3. M. H. L. Gao, C. Xu, "PDE-constrained optimization in aerodynamic shape design," Aerosp. Sci. Technol., vol. 112, p. 106640, 2021, doi: https://doi.org/10.1016/j.ast.2021.106640.
4. V. Kumar, S. K., Narayanaswamy, I., & Ramesh, "In Design and development of aerospace vehicles and propulsion systems: Proceedings of SAROD 2018," Springer Singapore, pp. 253–265, 2021, doi: https://doi.org/10.1007/978-981-15-9601-8_19.
5. D. E. Beckers, F. M., Schollenberger, M., Lutz, T., Bongen, D., Radespiel, R., Florenciano, J. L., & Funes-Sebastian, "CFD investigation of high-lift propeller positions for a distributed propulsion system," AIAA Aviat. 2022 Forum, p. 3217, 2022, doi: https://doi.org/10.2514/6.2022-3217.
6. A. V. Rao, K. S., Sivapragasam, M., Narahari, H. K., & Bharadwaj, "Aerodynamic optimization of transonic wing for light jet aircraft. In Design and development of aerospace vehicles and propulsion systems: Proceedings of SAROD 2018," Springer Singapore., pp. 253–265, 2021, doi: https://doi.org/10.1007/978-981-15-9601-8_19.
7. H. Zhang, J., & Wei, "A review on configuration optimization of hybrid energy system based on renewable energy," Heat Transf. data centers, vol. 61, p. 16648714, 2023, doi: https://doi.org/10.3389/fenrg.2022.977925.
8. N. R. Bombardieri, R., Cavallaro, R., Sanchez, R., & Gauger, "Aerostructural wing shape optimization assisted by algorithmic differentiation.," Struct. Multidiscip. Optim., 2020, doi: https://doi.org/10.1007/s00158-021-02884-5.
9. J. R. Yildirim, A., Kenway, G. K., Mader, C. A., & Martins, "A Jacobian-free approximate Newton–Krylov startup strategy for RANS simulations.," J. Comput. Phys., vol. 397, p. 108741, 2019, doi: https://doi.org/10.1016/j.jcp.2019.06.018.
10. G. Petrocchi, A., Mauriello, M., & Barakos, "Experimental and numerical investigation of transonic buffet and control-surface response," Aerosp. Sci. Technol., 2023, doi: https://eprints.gla.ac.uk/301258/1/301258.
11. D. A. Hoffmann, J., & Weiss, "Compressible and viscous effects in transonic planar flow around a circular cylinder-A numerical analysis based on a commercially available CFD tool," Fluids, vol. 8, no. 6, p. 182, 2023, doi: https://doi.org/10.3390/fluids8060182.
12. P. L. J. Wang, Y. Zhao, "Hybrid RANS–LES methods for transonic aerodynamic optimization," Fluids, vol. 9, no. 3, p. 68, 2024, doi: https://doi.org/10.3390/fluids9030068.

13. V. R. Aleksandr, N. E. T. K. A. C. H. E. V., & Kamireddi, “An overview on basics of computational fluid dynamics.,” J. Artif. Intell. Fluid Dyn., vol. 1, no. 1, pp. 21–30, 2022, doi: https://orcid.org/0000-0003-2710-8743.
14. W. J. Broadley, P., Nabawy, M. R., Quinn, M. K., & Crowther, “Dynamic experimental rigs for investigation of insect wing aerodynamics.,” J. R. Soc. Interface, vol. 19, no. 191, p. 20210909, 2020, doi: https://doi.org/10.1098/rsif.2021.0909.
15. Z. Shi, D., Pan, T., Zhu, X., & Li, “A strategy for predicting transonic compressor performance at low Reynolds number,” Aerosp. Sci. Technol., vol. 12, no. 4, p. 349, 2025, doi: https://doi.org/10.3390/aerospace12040349.
16. D. P. V. S. S. Dr M V D N S Madhavi1, Mir Sohail Ali2, Satyawan L. Dhondge3, Vijay Narayan Deshmukh4, Gajendra R. Gandhe5, “Advanced Numerical Methods For Solving Nonlinear Partial Differential Equations In Fluid Mechanics: Applications In Aerospace Engineering,” Int. J. Appl. Math., vol. 38, no. 3, pp. 140–152, 2025, doi: https://doi.org/10.12732/ijam.v38i3s.135.
17. M. L. Pinindriya, S. T., Soemaryanto, A. R., Fajar, M., Hidayat, K., Hamonangan, J. A., & Ramadiansyah, “Impact of airfoil section on winglet design for enhancing aerodynamics performance of aircraft using CFD analysis.,” Indones. J. Aerospace, vol. 5, no. 2, 2023, doi: https://doi.org/10.55981/ijoa.2023.900.
18. D. W. Yazdi, R. R., Reist, T. A., & Zingg, “Aerodynamic optimization of a Flying V aircraft based on the Reynolds-averaged Navier–Stokes equations.,” AIAA SCITECH 2024 Forum, vol. 19, no. 9, 2024, doi: https://doi.org/10.2514/6.2024-1909.
19. S. Wang, R. Wang, Y. Xia, Z. Sun, L. You, and J. Zhang, “Multi-objective aerodynamic optimization of high-speed train heads based on the PDE parametric modeling,” Struct. Multidiscip. Optim., pp. 1285–1304, 2021, doi: https://doi.org/10.1007/s00158-021-02916-0.
20. S. Champasak, P., Buaphet, P., & Bureerat, “Aeroelastic multi-objective optimisation of an aircraft wing taking into account gust alleviation,” SciTech Res. J., vol. 13, no. 2, pp. 1–10., 2021, doi: https://ph02.tci-thaijo.org/index.php/jstrmu/article/view/247300.
21. M. M. Dong, C., & Arief, “Morphing wing designs in commercial aviation,” arXiv Prepr. arXiv, p. 2502.07182, 2025, doi: https://arxiv.org/abs/2502.07182.
22. Q. S. Y. Chen, L. Zhang, “A review of morphing wing technology and applications,” Aerospace, vol. 9, no. 10, p. 574, 2022, doi: https://doi.org/10.3390/aerospace9100574.
23. R. Yu, S., Zhou, X., & Huang, “Time-varying aeroelastic modeling and analysis for a morphing wing.,” AIAA J., vol. 62, no. 10, pp. 3825-3840., 2024, doi: https://doi.org/10.2514/1.J063544.
24. G. Huang, C., Huang, J., Song, X., Zheng, G., & Yang, “Three dimensional aeroelastic analyses considering free-play nonlinearity using computational fluid dynamics/computational structural dynamics coupling,” J. Sound Vib., vol. 494, p. 115896, 2021, doi: https://doi.org/10.1016/j.jsv.2020.115896.
25. F. Wu, C. C., Völker, D., Weisbrich, S., & Neitzel, “The finite volume method in the context of the finite element method,” Mater. Today Proc., vol. 62, pp. 2679–2683, 2022, doi: https://doi.org/10.1016/j.matpr.2022.05.460.
26. S. Karahan, K., & Cadirci, “Investigation of fluid dynamics in various aircraft wing tank designs using 1D and CFD simulations,” Aerospace, vol. 11, no. 7, p. 519, 2024, doi: https://doi.org/10.3390/aerospace11070519.
27. D. (2021) Menter, F., Hüppe, A., Matyushenko, A., & Kolmogorov, “An overview of hybrid RANS–LES models developed for industrial CFD.,” Appl. Sci., vol. 11, no. 6, p. 2459, doi: https://doi.org/10.3390/app11062459.
28. X. Li, “Comprehensive Study and Practical Application of the Full Potential Equation in Addressing the Complexities of Transonic Flow Regimes. Science and Technology of Engineering,” Chem. Environ. Prot., vol. 1, no. 6, 2024, doi: https://doi.org/10.61173/04q1r742.

29. H. W. Y. Zhang, X. Liu, "Recent advances in CFD for transonic aerodynamics: A review," Aerospace, vol. 10, no. 2, p. 115, 2023, doi: https://doi.org/10.3390/aerospace10020115.
30. H. C. X. Li, Y. Wang, "Numerical investigation of transonic flow using CFD methods," Aerospace, vol. 10, no. 5, p. 412, doi: https://doi.org/10.3390/aerospace10050412.
31. S. Mohammad, M., Trounev, A., & Kumar, "High-precision Euler wavelet methods for fractional Navier–Stokes equations and two-dimensional fluid dynamics.," Phys. Fluids, vol. 36, no. 12, 2024, doi: https://doi.org/10.1063/5.0235658.
32. W. Sun and H. Li, "Efficient numerical methods for solving Navier–Stokes equations in complex geometries," J. Comput. Phys., vol. 472, p. 111678, 2023, doi: https://doi.org/10.1016/j.jcp.2022.111678.
33. J. J. Israr, A., Ali, Z. A., Alkhammash, E. H., & Jussila, "Optimization methods applied to motion planning of unmanned aerial vehicles: A review.," Drone, vol. 6, no. 5, p. 126, 2022, doi: https://doi.org/10.3390/drones6050126.
34. C. Otter, A., Murphy, J., Pakrashi, V., Robertson, A., & Desmond, "A review of modelling techniques for floating offshore wind turbines.," Wind Energy, vol. 25, no. 5, pp. 831–857, 2022, doi: https://doi.org/10.1002/we.2701.
35. J. R. Martins, "Aerodynamic design optimization: Challenges and perspectives," Comput. Fluids, vol. 239, p. 105391, 2022, doi: https://doi.org/10.1016/j.compfluid.2022.105391.
36. M. Wei, N. I. U., Zhang, Y., Chen, H. C. H., & Zhang, "Numerical study of a supercritical airfoil/wing with variable-camber technology," Chinese J. Aeronaut., vol. 33, no. 7, pp. 1850–1866, 2020, doi: https://doi.org/10.1016/j.cja.2020.01.008.